Exercise 10

Describe the following solids using inequalities. State the coordinate system used.

(a) A cylindrical shell 8 units long, with inside diameter 2 units and outside diameter 3 units

(b) A spherical shell with inside radius 4 units and outside radius 6 units

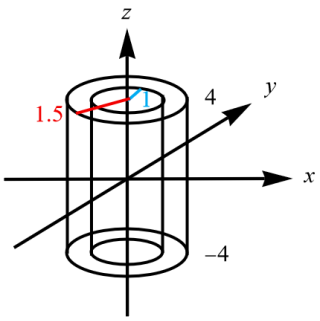
(c) A hemisphere of diameter 5 units

(d) A cube of side length 2

Solution

Part (a)

The region of interest is drawn below.

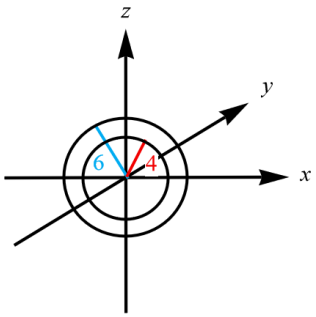


Use a cylindrical coordinate system (r, θ, z) centered on the cylinders' common axis at the midway point.

$$1 \le r \le 1.5, \qquad 0 \le \theta \le 2\pi, \qquad -4 \le z \le 4$$

Part (b)

The region of interest is drawn below.

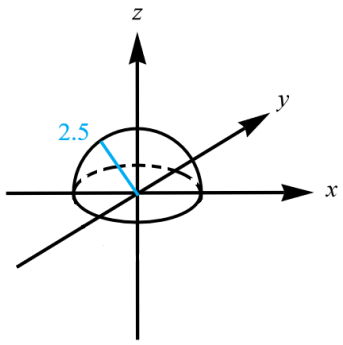


Use a spherical coordinate system (ρ, θ, ϕ) centered at the spheres' common center.

$$4 \le \rho \le 6, \qquad 0 \le \theta \le 2\pi, \qquad 0 \le \phi \le \pi$$

Part (c)

The region of interest is drawn below.

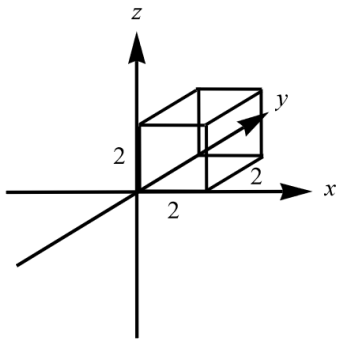


Use a spherical coordinate system (ρ, θ, ϕ) centered at the hemisphere's center.

$$0 \leq \rho \leq 2.5, \qquad 0 \leq \theta \leq 2\pi, \qquad 0 \leq \phi \leq \frac{\pi}{2}$$

Part (d)

The region of interest is drawn below.



Use a Cartesian coordinate system (x, y, z) centered at one of the cube's edges.

$$0 \leq x \leq 2, \qquad 0 \leq y \leq 2, \qquad 0 \leq z \leq 2$$